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RTCA Special Committee 186, Working Group 5

ADS-B UAT MOPS

Meeting #11

Third Draft of
Appendix M: UAT Error Detection and Correction Performance

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SUMMARY

This is the third draft of Appendix M of the UAT MOPS. This brief synopsis of the error detection and correction (EDAC) performance of the different UAT message types is an updated version of UAT-WP-10-02A.

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Appendix M

UAT Error Detection and Correction Performance

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This Appendix provides information on the performance of the Reed Solomon (RS) codes used by the various message types of UAT. The basic ADS-B message is a RS (30, 18) code word; the long ADS-B message is a RS(48,34) code word; and the ground up link message is six RS (92, 72) code words. These codes provide very strong error correction. Also, the error detection provided by these codes is sufficient to provide a maximum undetected error rate that is less than 10^{-8} for each of the message types, so additional CRC coding is not needed. Note that this excellent undetected error performance is due, in part, to the use of hard decision decoding. Schemes involving erasures might have considerably larger (i.e., degraded) undetected error rates.

The total word error rate for a RS (n, k) code is given by the formula

$$P_E = \sum_{j=t+1}^{n} \frac{n!}{j!(n-j)!} p_S^{\,j} (1-p_S)^{n-j},$$

where t=(n-k)/2 and p_S is the symbol error rate (SER). P_E includes both undetected and detected word error probabilities. Because there are 8 bits per symbol, the connection between the SER and the channel bit error rate (BER) is given by

$$p_S = 1-(1-p)^8 .$$

p is the channel BER.

The asymptotic value for the undetected word error rate (achieved when the channel bit error rate is 0.5) for a RS (n, k) code can be calculated using the formula

$$P_U = \frac{256^k - 1}{256^n} \sum_{j=0}^{t} \frac{n!}{j!(n-j)!} 255^j$$

where t=(n-k)/2. The results are given in Table M-1.

<u>Table M-1:</u> Maximum Undetected RS Word Error Rates

Code	Maximum Undetected Word Error Rate
RS(30,18)	2.06e-9
RS(48,34)	9.95e-10
RS(92,72)	5.74e-12

The undetected error performance of a RS code as a function of channel bit error rate can also be calculated, but the mathematical complexity is much greater [1]. The results are shown in Figures M-1 through Figure M-3. These graphs show total word error rate together with undetected word error rate. The detected word error rate, P_D, is just the difference between the two curves. If the correct word error rate is defined as P_C, then all the probabilities are related by

$$1 = P_E + P_C = P_U + P_D + P_C .$$

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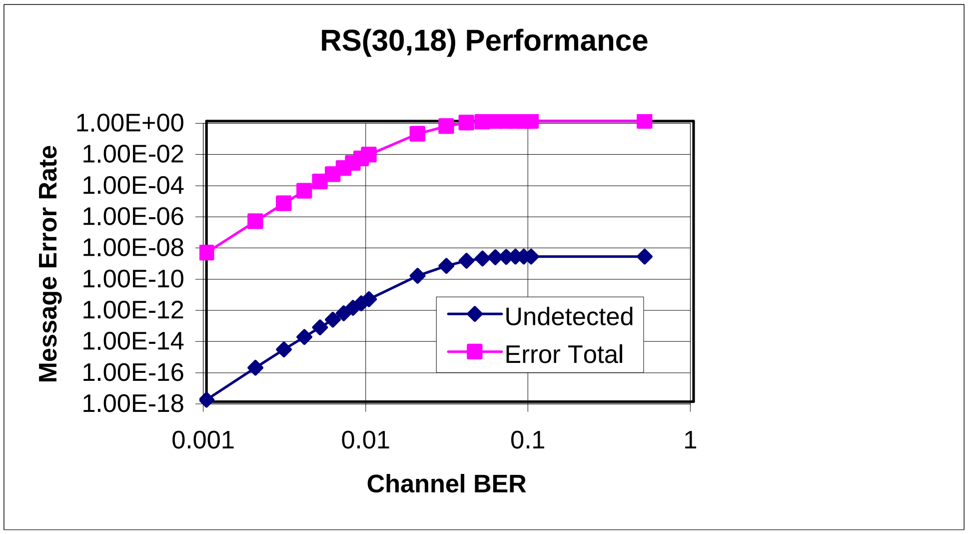

Figure M-1: Basic ADS-B Message Performance

(“Undetected” = P_U ; “Error Total” = P_E .)

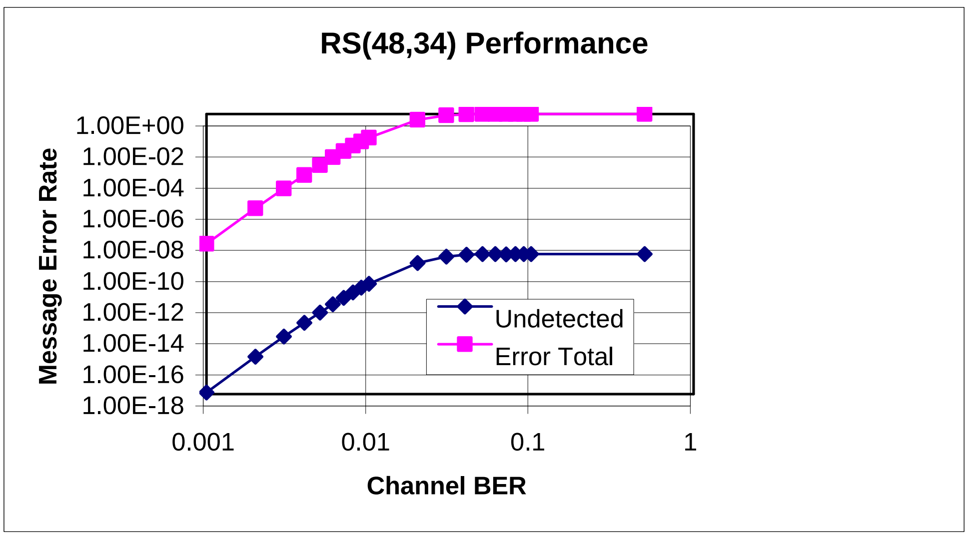

Figure M-2: Long ADS-B Message Performance

(“Undetected” = P_U ; “Error Total” = P_E .)

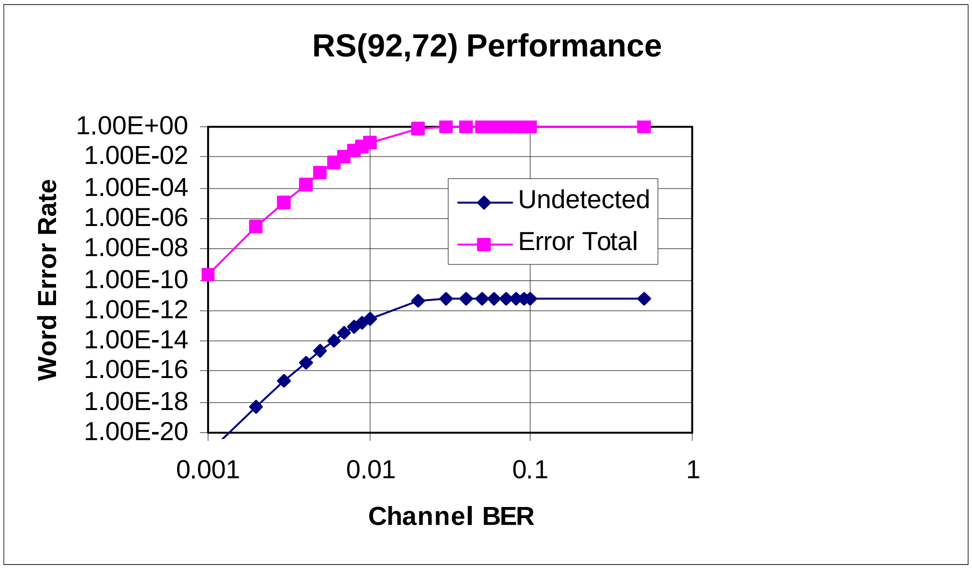

Figure M-3: Ground Up Link Message Performance

(“Undetected” = P_U ; “Error Total” = P_E .)

Note that for the ADS-B messages, the word error rate is equal to the message error rate because there is one word per message. This is not true for the ground up link message. Figure M-3 shows the performance of a single RS (92, 72) word. The performance of an entire message, consisting of six words, is given by

$$P_{Uburst} = (1 - P_E + P_U)^6 - (1 - P_E)^6 = (P_C + P_U)^6 - P_C{}^6$$

and

$$P_{Eburst} = 1 - (1 - P_E)^6 = 1 - P_C{}^6$$

Again, P_E is the total word error rate, and P_U is the undetected word error rate. A graph of the undetected message error rate versus the channel BER is shown in Figure M-4. This figure indicates that the maximum undetected error rate is about 1.3e-12, which occurs when the channel BER is about 0.012. To see why there is a maximum, consider the following approximation:

$$P_{UBurst} \approx 6 P_U (1 - P_E)^5 .$$

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The P_U term is small at low BER and the $(1-P_E)^5$ term is small at high BER (because P_E is nearly 1 in that case).

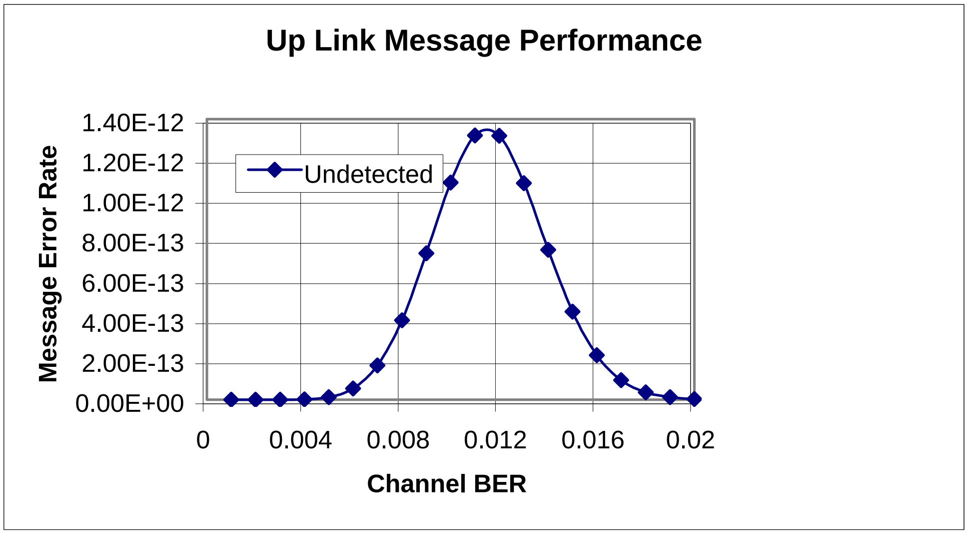

Figure M-4: Ground Up Link Message Undetected Message Error Rate

("Undetected" = P_{UBurst} .)

For completeness, a graph of the total up link message error rate versus channel BER is also provided in Figure M-5.

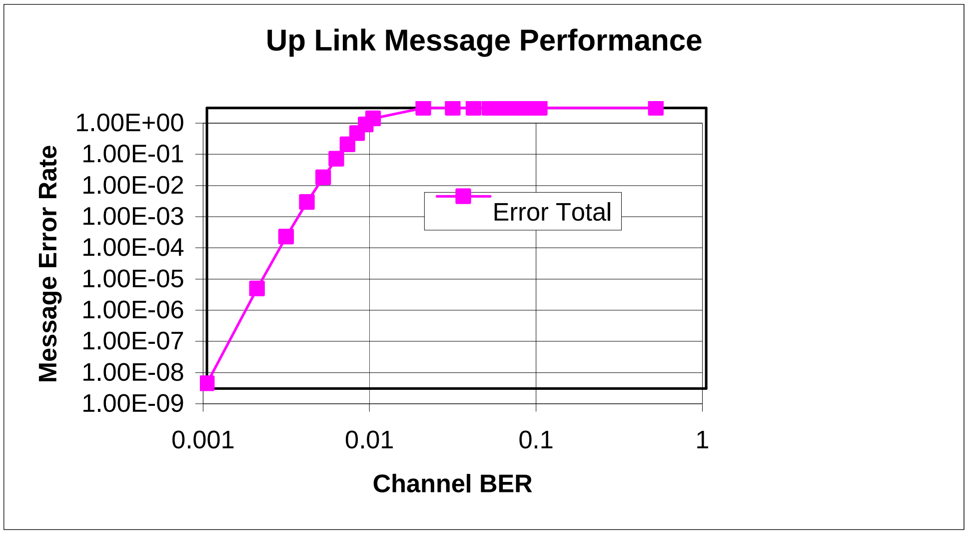

Figure M-5: Ground Up Link Message Total Message Error Rate

("Error Total" = P_{EBurst} .)

Reference [1]: Kasami,T., and S. Lin, 1984, "On the Probability of Undetected Error for Maximum Distance Separable Codes," IEEE Trans. Comm., COM-32,998-1006.

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